CSC 120 Lab 00

**(+30)** Developing **an** optimal strategy for a variant of the game *Nim****. The problems appear at the end***

Nim is a subtraction game that is played with sticks. The subtraction game variant is simple. A pile of sticks is placed in front of a pair of participants. The players take turns removing either 1, 2, 3, or 4 sticks from the pile. **The player who removes that last stick from the pile loses the game**. It turns out that there is an optimal strategy for playing this subtraction game variant of *Nim*. The purpose of this exercise is to find the strategy (solution.)

### Rules of the game

We begin by considering the rules of the game. A player loses the game if he/she is forced to pick up the last stick in the pile. Thus, a pile containing a single stick is **bad pile**. Other piles of sticks are not so bad. Consider a pile that contains 2 sticks. If it is your turn and you have a pile with 2 sticks then you can pick up a single stick which will leave your opponent with a bad pile containing a single stick. Likewise, if it is your turn and you have a pile with 3 sticks then you can pick up 2 sticks which will leave your opponent with a bad pile containing a single stick. And if it is your turn and you have a pile with 4 sticks then you can pick up 3 sticks which will leave your opponent with a bad pile containing a single stick. Finally, if it is your turn and you have a pile with 5 sticks then you can pick up 4 sticks which will leave your opponent with a bad pile containing a single stick.

Number of sticks **Your turn** Outcome

1 no strategy you lose (bad pile)

2 remove 1 stick you win

3 remove 2 sticks you win

4 remove 3 sticks you win

5 remove 4 sticks you win

6 no strategy you lose (bad pile)

7 remove 1 you win

…

11 no strategy you lose

### Why it is a bad pile for you with 6 sticks

Note that if it is your turn and you a have a pile with 6 sticks then there is nothing you can do to prevent your opponent from giving you a bad pile after his/her turn. If you take a single stick then he/she can take 4 sticks, leaving you with a bad pile. If, on the other hand, you take 2 sticks then he/she can take 3 sticks, leaving you with a bad pile. If your take 3 sticks then he/she can take 2 sticks, leaving you with a bad pile. Finally, if you take 4 sticks then he/she can take a single stick, leaving you with a bad pile. So, a **pile with 6 sticks is just as bad as a pile with a single stick**.

### Why it is a good pile for you with 7, 8, 9, 10 sticks

A pile with 7 sticks, on the other hand, is great because you can take a single stick and force your opponent to have to deal with a **bad pile containing 6 sticks**. Likewise, you can force your opponent to have to deal with a bad pile containing 6 sticks if you have a pile with 8, 9, or 10 sticks by removing 2, 3, or 4 sticks, respectively. A pattern is clearly arising.

### Why it is a bad pile for you with 11 sticks

Note that if it is your turn and you a have a pile with 11 sticks then there is nothing you can do to prevent your opponent from giving you a bad pile after his/her turn. If you take a single stick then he/she can take 4 sticks, leaving you with a bad pile (6 sticks). If, on the other hand, you take 2 sticks then he/she can take 3 sticks, leaving you with a bad pile(6 sticks). If your take 3 sticks then he/she can take 2 sticks, leaving you with a bad pile (6 sticks).. Finally, if you take 4 sticks then he/she can take a single stick, leaving you with a bad pile(6 sticks).. So, a **pile with 11 sticks is just as bad as a pile with a 6 sticks**.

## Problems

1. (+5) As pointed out, you lose if the pile has 1, 6, 11 or 16 sticks. How many sticks are in the next two piles for you to lose

Discussion that will assist in problems 2 3 4 5

n = 3k + 1 with k = 0, 1, 2, 3, 4, etc. which results in the set {1, 4, 7, 10, 13 …}

QUESTION how can we determine if n = 321 is in the above set?

ANSWER we need to find a k with k == 0,1,2, 3 etc. such that k has to be an integer

321 = 3k + 1

320 = 3k

k = 320/3

k = 106.666 hence we can conclude 321 is NOT in the set above

QUESTION Is n = 205 in the above set?

ANSWER

Yes since 205 = 3\*68 + 1

1. (+10) ***Given the pattern n = 4k - 3 with k = 1, 2, 3, 4, etc. results in the set { 1, 5, 9, 13 … }***

Is n = 195 in the set ? **Explain**

195 = 4k – 3

198 = 4 k

K = 198/4

K = 49.5 since k= 49.5 which is not in the set, so n=195 is not in the set

Is n = 237 in the set ? **Explain**

237 = 4k – 3

K = 240 / 4

K = 60 because k in the set so n = 237 in the set also.

237 = 4 \* 60 - 3

1. (+5) Identify the pattern for our game of Nim . For a pile containing *n* sticks, which ones are bad for you? Express the pattern in the most general way possible using n = number of sticks in the pile Your answer should include n and k. See discussion above

***n = ??? with k = 0,1,2,3 should result in the set { 1, 6, 11, 16, etc }***

n= 1, 6 , 11 are the bad ones

n = 5k +1 to get the set { 1, 6, 11, 16, etc }

1. (+5) Is a pile with 286 sticks a good pile or a bad pile for you? **Explain Using your pattern**

286 = 5 k +1

285 = 5 k

K = 57

286 = 5 \* 57 +1 which is bad pile since it is included in the set of the pile.

1. (+5) Is a pile with 285 sticks a good pile or a bad pile for you? **Explain Using your pattern**

285 = 5 k +1

284 = 5 k

K = 56.5 which Is not included in the set of the pad pile, so it is good pile.